MATH 1B FINAL (LEC 002) PROFESSOR PAULIN

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

CALCULATORS ARE NOT PERMITTED

THIS EXAM WILL BE ELECTRONICALLY SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS IN THE SPACES PROVIDED. YOU MAY WRITE SOLUTIONS ON THE BLANK PAGE AT THE BACK BUT BE SURE TO CLEARLY LABEL THEM

Name: _____

Student ID: _____

GSI's name: _____

This exam consists of 10 questions. Answer the questions in the spaces provided.

1. (30 points) Compute the following integrals:

(a)

 $\int \arctan(x) dx$

(b)

$$\int \frac{(\sqrt{x^2-1})^3}{x^8} dx$$

You should express your final answer in terms of x using an appropriate right triangle.

2. (30 points) Determine if the following series are absolutely convergent, conditionally convergent or divergent. You do not need to show your working.

(a)

$$\sum_{n=2}^{\infty} \log_2(n+1) - \log_2(n-1)$$

Solution:

(b)

$$\sum_{n=1}^{\infty} \frac{\sqrt{n^4 - 1}}{n^3 + 3}$$

Solution:

(c)

$\sum_{k=1}^{\infty}$	$\sin(\pi n)$
$\sum_{n=1}$	n

Solution:

(d)

$$\sum_{n=1}^{\infty} \frac{2^n + 4^n + 5^n}{6^n - 4^n}$$

Solution:

(e)

$$\sum_{n=2}^{\infty}(-1)^nn\tan(\frac{1}{n^2})$$

3. (30 points) Determine the interval of convergence of the following power series

$$\sum_{n=1}^{\infty} \frac{(2x+1)^{3n}}{8^n \sqrt[3]{n+1}}.$$

Carefully justify your answer.

4. (30 points) Calculate the Maclaurin Series of the following function.

$$f(x) = (x-1)^2 \sin(x).$$

Express your final answer in sigma-notation.

What is the value of $f^{(100)}(0)$?

5. (30 points) Find a general solution to the following differential equation

$$x^2y' = e^x - 4xy, \quad x > 0.$$

6. (30 points) Find an equation for the orthogonal trajectory to the family of curves

$$y = 2 + \sqrt[3]{x^2 + k}$$
 (k any constant)

which contains the point (3, 2).

7. (30 points) Give an example of a differential equation y' = F(y) with the property that $\lim_{x\to\infty} y(x) = 1$ for any initial condition $y(0) = y_0$. Give a precise formula and draw the graph of y' versus y. Justify your answer with the method of direction fields.



8. (30 points) Are there any solutions to

$$y'' - 6y' + 10y = 0$$

satisfying the conditions y(0) = 0 and $y(\pi) = 1$? Carefully justify your answer. Solution: 9. (30 points) Find a general solution to the following differential equation.

$$y'' - y = 2(3x^2 + 3x + 1)e^x$$

10. (30 points) Find a power series (centered at 0) which is a solution to

$$y'' = 2xy' + 2, \ y(0) = 0, y'(0) = 0.$$