

School of Mathematics and Applied Statistics

## Student to complete:

Family name	
Other names	
Student number	

## MATH221 Mathematics for Computer Science Wollongong and Southwest Sydney Campuses

## Final Examination Paper Autumn Session 2021

Exam duration	3 hours
Items permitted by examiner	UOW-approved calculator and all instructor-supplied subject material
Aids supplied	None

Instructions:

- Show all your work. Answers given without reasonable explanation will receive a mark of zero.
- The exam is open-book; this means you are allowed to refer to the lecture notes and all other resources supplied by the instructor. You are not allowed to use any software or on-line resource for assistance. A calculator is permitted.
- You have three hours to complete the exam, plus 20 minutes to scan and convert your work to pdf format and submit it on Moodle. Only pdf documents are accepted.
- The Zoom classroom will be open and the instructor will be there during the exam. You are not required to be in the room; it is optional.

$$(p \land q) \lor [\sim p \lor (p \land \sim q)]$$

a tautology, fallacy or contingent statement? Give a proof for your answer, either by truth table or by quick method.

Question 2 (5 marks). Solve the equation

$$x = 3^{50} \pmod{77}, \ 0 \le x < 77.$$

Question 3 (3 marks). Disprove the statement p:

$$p: \forall x \in \mathbb{R} \exists y \in \mathbb{R} \text{ s.t. } \frac{2x}{3y} = 1$$

by writing the negation  $\sim p$  and proving  $\sim p$ .

**Question 4 (5 marks).** On  $A = \{a, b, c, d\}$ , define a relation R as follows:

 $R = \{(d, d), (a, b), (c, b), (b, b), (a, d), (c, a), (a, a), (c, c), (a, c), (b, a), (d, a), (d, b)\}.$ 

Prove or disprove that R is an equivalence relation.

Question 5 (8 marks). Use the Euclidean algorithm to find gcd(5355, 3009).

**Question 6 (8 marks).** Define a binary operation \* on  $\mathbb{R}$  by

 $a \ast b = 1 + ab + a.$ 

Determine whether this operation is commutative and/or associative.

Question 7 (4 marks). Let  $T = \{-2, -1, 0, \frac{1}{3}, \frac{2}{3}, \frac{1}{2}, 1, 2, 3, 4\}.$ 

- (a) Which elements of T, if any, are invertible under multiplication?
- (b) Which elements of T, if any, are invertible under addition?

Question 8 (5 marks). Using the equivalence classes modulo 25, find a value of x < 0 that satisfies

[x][7] = [1],

or explain why one does not exist.

Question 9 (4 marks). Let  $P = \{a, b, c\}$  and  $Q = \{0, 1, 2, 3\}$ .

- (a) Define a function from P to Q that is injective and not surjective, or explain why that is not possible.
- (b) Define a function from P to Q that is surjective and not injective, or explain why that is not possible.

**Question 10 (4 marks).** The following animals need to be loaded single-file into a trailer: a horse, a cow, a chicken, a goat and a fox. How many different ways are there to accomplish this, such that the chicken and the fox are not placed beside each other?

Question 11 (5 marks). The following statistics about MATH221 are known.

- 1. 50% of all MATH221 students study hard.
- 2. Of the ones that do not study hard, 90% of them do not pass the final exam.
- 3. Of the ones that study hard, only 10% of them do not pass the final exam.

Use a Venn diagram to answer the question: what is the probability that a randomly selected MATH221 student studies hard and passes the final exam?

**Question 12 (5 marks).** Draw a complete bipartite graph that has at least 7 vertices in total. Assign positive weights to all edges such that no two edges have the same weight. Find the minimum spanning tree and its weight.