Quiz 1

ⓘ This is a preview of the published version of the quiz.

Started: Nov 8 at 18:55

Quiz Instructions

Question 1

1 pts

Let$X_1, X_2 . . . . , X_n$ be identically and independently distributed random variables, each with mean 0, variance 1, and moment generating function $M_X(t)$. Define the following quantities

$S_n = \sum_{i=1}^n X_i$, $\bar{X}_n = \frac{1}{n}\sum_{i=1}^n X_i$, $Z_n = n^{1/2}\bar{X}_n$. Which of the following statement is **not correct?**

- $M_{Z_n}(t) = \left\{M_{S_n}\left(\frac{t}{\sqrt{n}}\right)\right\}^n$
- $M_{\bar{X}_n(t)} = \left\{M_X\left(\frac{t}{n}\right)\right\}^n$
- $Z_n \xrightarrow{d} N(0,1)$ when $n \to \infty$.
- $M_{S_n(t)} = \{M_X(t)\}^n$

Question 2

1 pts

Consider the two random variables X and Y that have a bivariate normal distribution with mean $\mu_x = \mu_y = 0$, standard deviation $\sigma_x = \sigma_y = 1$ and correlation ρ. What is the value of $E(XY^2)$ (hint: using conditional expectation)?

- ρ
- 0

○ 1

○ ρ^2

Question 3

1 pts

Consider tossing a six-sided die 20 times, and suppose the die is unbalanced so that the probability of observing an i in each toss is $i/21$. Let X_i be the number of times i appears out of 20 tosses. What is the distribution of $X_1 + X_3 + X_4$? You can just give the distribution without showing any work. (Hint: using intuition)

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Question 4

1 pts

Consider the two discrete random variables X and Y, each of which only receives a value of 1 or -1. Suppose the marginal distribution of X is given by $P(X = 1) = 0.7,\ P(X = -1) = 0.3.$ The conditional expectation

$E(Y|X=1) = 0.6,\ E(Y|X=-1) = -0.5$. What is the value for the marginal expectation $E(Y)$?

For questions 5-8, please show all the workings. Upload your work to the corresponding **Assignment item**. Make sure you upload your work by 3:05pm (Wednesday 31 Aug - Sydney time).

Question 5 (1.5 marks)

Let U be a continuous uniform random variable on $(0, 1)$. Find the density for $V = 1/U$. Make sure to specify the range of values (i.e the support) of V.

Question 6 (1.5 marks)

Consider tossing a coin n times; for each time, the probability that a coin lands head is p. The number of heads, N, is recorded. Then, the same coin is tossed for N more times. What is the expectation for the total number of heads in this process?

Question 7 (3 marks)

Suppose the bivariate random variables X and Y have the joint density

$$f_{X,Y}(x,y) = \frac{1}{\sqrt{2\pi x^2}} \exp\left\{-\frac{1}{2x^2}(y-x)^2\right\},\ 0 < x < 1, -\infty < y < \infty$$

Let $U = \frac{Y}{X}$. Find the joint density of U and X and show that U and X are independent.

Question 8 (2 marks)

Consider the hierarchical model $Y|N \sim \chi^2_{2N},\ N \sim \mathrm{Poisson}(\theta)$.

(a) Show that the (marginal) moment generating function of Y is given by $M_Y(t) = \exp\left(\frac{2t\theta}{1-2t}\right),\ t < 1/2$. You may find the expression $a^b = e^{b\log(a)}$ useful for the proof.

(b) Using the fact that $E(Y) = 2\theta,\ \mathrm{Var}(Y) = 8\theta$, prove that as $\theta \to \infty$, the random variable $Z = \frac{Y - E(Y)}{\sqrt{\mathrm{Var}(Y)}} \xrightarrow{d} N(0,1)$.

Some useful formulas: **Useful formulas.pdf**

(https://canvas.sydney.edu.au/courses/44955/files/26270143/download?download_frd=1)

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